5) $dU = TdS - PdV + \mu dN$ (1)

2/2

$$T = 2(\theta/R)s = \frac{2\theta}{R}\frac{S}{N}$$
$$P = 2(R\theta/V_0^2)\cancel{V} = 2\left(\frac{R\theta}{V_0^2}\right)\frac{V}{N}$$
(2)

$$dU = \frac{2\theta}{R}\frac{S}{N}dS - 2\frac{R\theta}{V_0^2}\frac{V}{N}dV + \mu dN \quad (3)$$

$$= \left(\frac{\partial U}{\partial S}\right)_{V,N} dS + \left(\frac{\partial U}{\partial V}\right)_{S,N} dV + \left(\frac{\partial U}{\partial N}\right)_{S,V} dN \quad (4)$$

$$U = \int \frac{2\theta}{R}\frac{S}{N}dS + f(V,N)$$
$$= \frac{\theta}{R}\frac{S^2}{N} + f(V,N) \quad (5)$$

$$U = -\int \frac{2R\theta}{V_0^2}\frac{V}{N}dV + g(S,N)$$
$$= -\frac{R\theta}{V_0^2}\frac{V^2}{N} + g(S,N) \quad (6)$$

From (5):

$$\left(\frac{\partial U}{\partial V}\right)_{S,N} = \left(\frac{\partial f}{\partial V}\right)_{S,N} = -\frac{2R\theta}{V_0^2}\frac{V}{N} \quad (7)$$

From (6):

$$\left(\frac{\partial U}{\partial S}\right)_{V,N} = \left(\frac{\partial g}{\partial S}\right)_{V,N} = \frac{2\theta}{R}\frac{S}{N} \quad (8)$$

from (7) & (8):

$$f = \frac{-R\theta}{V_0^2}\frac{V^2}{N} + f'(N)$$

$$g = \frac{\theta}{R}\frac{S^2}{N} + g'(N)$$

$$\left.\begin{aligned} &f = \frac{-R\theta}{V_0^2}\frac{V^2}{N} + f'(N) \\ &g = \frac{\theta}{R}\frac{S^2}{N} + g'(N)\end{aligned}\right\} (9)$$

From (9) & (5)/(6):

$$u = \frac{\theta S^2}{RN} - \frac{R\theta}{V_0^2}\frac{V^2}{N} + f'(N) \qquad (10)$$

$$u = -\frac{R\theta}{V_0^2}\frac{V^2}{N} + \frac{\theta}{R}\frac{S^2}{N} + g'(N) \qquad (11)$$

from (10) & (11):

$$f'(N) = g'(N) = \text{constant} \equiv u_0$$

$\Rightarrow$

$$u = \frac{\theta}{RN}S^2 - \frac{R\theta}{V_0^2}\frac{V^2}{N} + Nu_0 \qquad (12)$$

#

b) $\mu = \left(\frac{\partial u}{\partial N}\right)_{S,V}$

$$= -\frac{\theta}{R}\frac{S^2}{N^2} + \frac{R\theta}{V_0^2}\frac{V^2}{N^2}$$

$$\mu = \left(\frac{R}{V_0^2}V^2 - \frac{S^2}{R}\right)\frac{\theta}{N^2} + \mu_0 \qquad (13)$$

#

c)

$$TS = \frac{2\theta}{R}\frac{S^2}{N}$$

$$PV = \frac{2R\theta}{V_0^2}\frac{V^2}{N}$$

$$\mu N = \left(\frac{RV^2}{V_0^2} - \frac{S^2}{R}\right)\frac{\theta}{N}$$

$$TS - PV + \mu N$$

$$= \frac{2\theta}{R}\frac{S^2}{N} - \frac{2R\theta}{V_0^2}\frac{V^2}{N} + \frac{R\theta}{V_0^2}\frac{V^2}{N} - \frac{\theta}{R}\frac{S^2}{N}$$

$$= \frac{\theta}{R}\frac{S^2}{N} - \frac{R\theta}{V_0^2}\frac{V^2}{N}$$

$$= U$$ # Euler equation is satisfied!