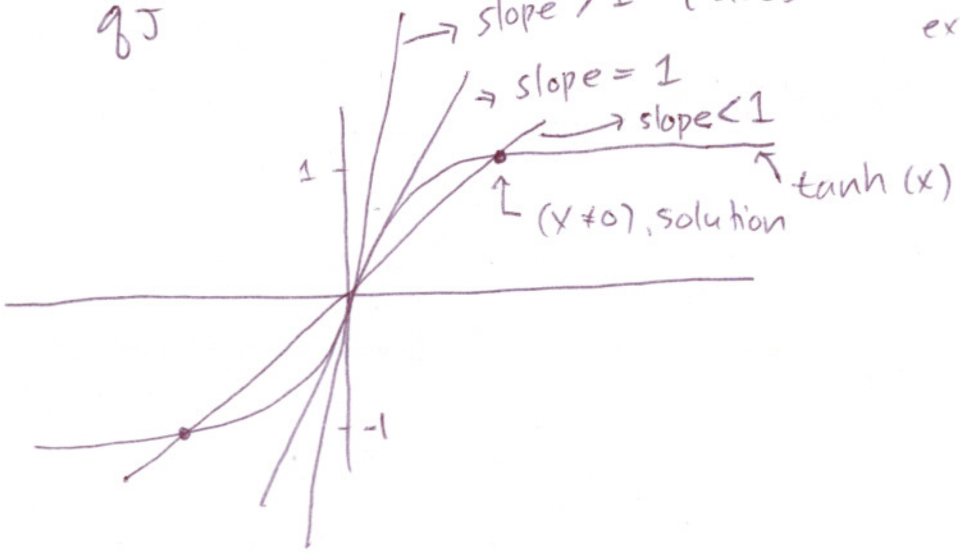
40) $$m = \tanh(\beta\mu^* B + \beta q J m) \qquad (1)$$

2|2

$$x \equiv \beta q J m \qquad (2)$$

a) For $B = 0$,

$$\frac{kTx}{qJ} = \tanh x \qquad (3)$$



$$\frac{kT_c}{qJ} = 1$$ (slope of the RHS of (3) at the onset of having a solution for $x \neq 0$)

$$T_c = \frac{qJ}{k} \qquad (4)$$ #

b) for $B = 0$:

$$m = \frac{\exp(\beta q J m) - \exp(-\beta q J m)}{\exp(\beta q J m) + \exp(-\beta q J m)}$$

$$= \frac{\exp(2\beta q J m) - 1}{\exp(2\beta q J m) + 1}$$

$$\Rightarrow \exp(2\beta q J m) = \frac{1+m}{1-m}$$

$$\beta q J = \frac{1}{2m} \ln\left[\frac{1+m}{1-m}\right] \quad , \quad \beta q J = \frac{T_c}{T}$$

$$\frac{T_c}{T} = \frac{1}{2m} \ln\left(\frac{1+m}{1-m}\right) \quad \# \qquad (5)$$

c) $$\ln(1-m) = -\int \frac{dm}{(1-m)} = -\int \left(\sum_n m^n\right) dm$$

$$= -\sum_n \frac{m^{n+1}}{n+1} \qquad (6)$$

similarly,

$$\ln(1+m) = \sum_n (-1)^n \frac{m^{n+1}}{n+1} \qquad (7)$$

(6), (7) → (5):

$$\frac{T_c}{T} = \frac{1}{2} \sum_n \left\{ \frac{(1+(-1)^n)\, m^n}{n+1} \right\}$$

$$= \sum_{\text{even}} \frac{m^n}{n+1} = \sum_{n=0}^{\infty} \frac{m^{2n}}{(2n+1)} \quad \# \qquad (8)$$

At $B=0$, (2) is an even function of m because it has no preferred direction for the spontaneous magnetic field.

d) for $|m| \ll 1$

$$\frac{T_c}{T} \simeq 1 + \frac{m^2}{3}$$

$$\Rightarrow \frac{T}{T_c} \simeq 1 - \frac{m^2}{3}$$

$$m^2 = 3\left(\frac{T_c - T}{T_c}\right)$$

$$m = \sqrt{3}\left(\frac{T_c - T}{T_c}\right)^{1/2}$$

$$m \propto \left[\frac{(T_c - T)}{T_c}\right]^{\beta}$$

where $\beta = 1/2$

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