3) $$\alpha_s = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_{S,N} = \frac{1}{V}\frac{\partial(V,S,N)}{\partial(T,P,N)}\frac{\partial(T,P,N)}{\partial(T,S,N)}$$

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$$= \frac{1}{V}\left[\left(\frac{\partial V}{\partial T}\right)_{P,N}\left(\frac{\partial S}{\partial P}\right)_{T,N} - \left(\frac{\partial V}{\partial P}\right)_{T,N}\left(\frac{\partial S}{\partial T}\right)_{P,N}\right]\left(\frac{\partial P}{\partial S}\right)_{T,N} \quad (1)$$

but $V\alpha = \left(\frac{\partial V}{\partial T}\right)_{P,N}$

$$\left.\begin{aligned} &\left(\frac{\partial S}{\partial P}\right)_{T,N} = -\left(\frac{\partial V}{\partial T}\right)_{P,N} = -V\alpha = \frac{1}{\left(\frac{\partial P}{\partial S}\right)_{T,N}} \\ &\left(\frac{\partial V}{\partial P}\right)_{T,N} = -K_T V \\ &\left(\frac{\partial S}{\partial T}\right)_{P,N} = C_P/T \end{aligned}\right\} \quad (2)$$

(2) → (1):

$$\alpha_S = \frac{1}{V}\left[-\alpha^2V^2 + \frac{VK_TC_P}{T}\right]\frac{-1}{V\alpha}$$

$$= \frac{-1}{V^2\alpha}\left[-\alpha^2V^2 + \frac{VK_TC_P}{T}\right]$$

$$= \alpha\left(1 - C_P\frac{K_T}{\alpha^2VT}\right) \quad \text{but } \frac{K_T}{\alpha^2VT} = \frac{1}{C_P - C_V}$$

$$= \alpha\left(1 - \frac{C_P}{C_P - C_V}\right)$$

$$\Rightarrow \frac{\alpha_S}{\alpha} = \frac{C_V}{C_V - C_P} = \frac{1}{1-\gamma}, \text{ where } \gamma = C_P/C_V \quad (3)$$

b) Monoatomic ideal gas:

$$\alpha = \frac{1}{T}, \quad C_V = \frac{3}{2}NR, \quad C_P = \frac{5}{2}NR \quad (4)$$

(4) → (3):

$$\alpha_s = \frac{1/T}{1-5/3} = \frac{-3}{2T} \quad \#$$ (4)

From problem 2:

$$pV^{5/3} = \text{constant}$$

$$\frac{NRT}{V} V^{5/3} = \text{constant}$$

$$\Rightarrow \quad TV^{2/3} = \text{constant} \quad , N,R \text{ are constants}$$

$$V = \frac{K}{T^{3/2}} \quad , \text{ where } K \text{ is a constant}$$

$$\alpha_s = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_{S,N}$$

$$= \frac{1}{V}\frac{\partial}{\partial T}\left(\frac{K}{T^{3/2}}\right) = \frac{T^{3/2}}{K} K \frac{\partial}{\partial T}\left(T^{-3/2}\right)$$

$$\alpha_s = -\frac{3}{2} T^{3/2} T^{-5/2} = -\frac{3}{2}\frac{1}{T} \quad \#$$

c) Since S is held constant, V must decrease as T is increased. Thus, α_s is negative.