

8.28) For $f(x) = \sum_{k=0}^n a_k x^k$

$$\tilde{f}(u) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-iux} f(x) dx$$

$$= \frac{1}{\sqrt{2\pi}} \sum_{k=0}^n a_k \int_{-\infty}^{+\infty} e^{-iux} x^k dx$$

$$= \frac{1}{\sqrt{2\pi}} \sum_{k=0}^n a_k \left(\frac{1}{-i}\right)^k \frac{d^k}{du^k} \left(\int_{-\infty}^{+\infty} e^{-iux} dx \right)$$

$$= \frac{1}{\sqrt{2\pi}} \sum_{k=0}^n a_k (i)^k \frac{d^k}{du^k} (2\pi \delta(u))$$

note: $\delta(x-x') = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{i(x-x')t} dt$

or $\delta(u) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} dk e^{iuk}$

$$\tilde{f}(u) = \sqrt{2\pi} \sum_{k=0}^n i^k a_k \delta^{(k)}(u)$$

, where $\delta^{(k)}(u) \equiv \frac{d^k}{du^k} \delta(u)$