2.26 $|a_1\rangle = \vec{a}_1 = (1,1,-1) \Rightarrow |\bar{a}_1\rangle = \frac{1}{\sqrt{3}}(1,1,-1)$
$|a_2\rangle = \vec{a}_2 = (-2,1,-1) \Rightarrow |\bar{a}_2\rangle = \frac{1}{\sqrt{6}}(-2,1,-1)$ } normalized

a) $P_1 = |\bar{a}_1\rangle\langle\bar{a}_1|$

$$= \frac{1}{3}\begin{pmatrix}1\\1\\-1\end{pmatrix}(1\ 1\ -1) = \frac{1}{3}\begin{pmatrix}1&1&-1\\1&1&-1\\-1&-1&1\end{pmatrix}$$

$$P_1^2 = \frac{1}{9}\begin{pmatrix}1&1&-1\\1&1&-1\\-1&-1&1\end{pmatrix}\begin{pmatrix}1&1&-1\\1&1&-1\\-1&-1&1\end{pmatrix} = \frac{1}{9}\begin{pmatrix}3&3&-3\\3&3&-3\\-3&-3&3\end{pmatrix} = \frac{1}{3}\begin{pmatrix}1&1&-1\\1&1&-1\\-1&-1&1\end{pmatrix} = P_1$$

$P_2 = |\bar{a}_2\rangle\langle\bar{a}_2|$

$$= \frac{1}{6}\begin{pmatrix}-2\\1\\-1\end{pmatrix}(-2\ 1\ -1) = \frac{1}{6}\begin{pmatrix}4&-2&2\\-2&1&-1\\2&-1&1\end{pmatrix}$$

$$P_2^2 = \frac{1}{6^2}\begin{pmatrix}4&-2&2\\-2&1&-1\\2&-1&1\end{pmatrix}\begin{pmatrix}4&-2&2\\-2&1&-1\\2&-1&1\end{pmatrix} = \frac{1}{6^2}\begin{pmatrix}24&-12&12\\-12&6&-6\\12&-6&6\end{pmatrix} = \frac{1}{6}\begin{pmatrix}4&-2&2\\-2&1&-1\\2&-1&1\end{pmatrix} = P_2$$

b) $$P_1 + P_2 = P = \frac{1}{3}\begin{pmatrix}1&1&-1\\1&1&-1\\-1&-1&1\end{pmatrix} + \frac{1}{3}\begin{pmatrix}2&-1&1\\-1&1/2&-1/2\\1&-1/2&1/2\end{pmatrix} = \frac{1}{3}\begin{pmatrix}3&0&0\\0&3/2&-3/2\\0&-3/2&3/2\end{pmatrix} = \begin{pmatrix}1&0&0\\0&1/2&-1/2\\0&-1/2&1/2\end{pmatrix}$$

$$P^2 = \begin{pmatrix}1&0&0\\0&1/2&-1/2\\0&-1/2&1/2\end{pmatrix}\begin{pmatrix}1&0&0\\0&1/2&-1/2\\0&-1/2&1/2\end{pmatrix} = \begin{pmatrix}1&0&0\\0&1/2&-1/2\\0&-1/2&1/2\end{pmatrix} = P$$

c) $$P(r) = \begin{pmatrix}1&0&0\\0&1/2&-1/2\\0&-1/2&1/2\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}x\\ 1/2\,y - 1/2\,z\\ -1/2\,y + 1/2\,z\end{pmatrix}$$

$$|a_3\rangle \equiv \vec{a}_1 \times \vec{a}_2 = \begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&1&-1\\-2&1&-1\end{vmatrix} = \begin{pmatrix}0\\3\\3\end{pmatrix}$$

$$\langle Pr|a_3\rangle = 3(1/2\,y - 1/2\,z) + 3(-1/2\,y + 1/2\,z) = 0$$

$\Rightarrow$ P projects a vector into the plane spanned by $\vec{a}_1$ ~~...~~ & $\vec{a}_2$