

1.22 Find $f: \mathbb{R}^2 \rightarrow \mathbb{R}$

$f(\alpha|a\rangle) = \alpha f(|a\rangle)$ but f is not linear

$\alpha \in \mathbb{R}, |a\rangle \in \mathbb{R}^2$

$$f|a\rangle \equiv \sqrt{\langle a|a\rangle}$$

thus,

$$f(\alpha|a\rangle) = \sqrt{\langle \alpha a | \alpha a \rangle} = \sqrt{\alpha^2 \langle a | a \rangle} = \alpha f|a\rangle$$

but, $f(|a\rangle + |b\rangle) = \|a+b\| \leq \|a\| + \|b\|$

equality holds only if $a=b$, thus f is not linear

comment: this example is problematic if $\alpha < 0$