30/30

2) Jackson 7.22

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a) $$\mathrm{Im}\,\frac{\varepsilon(\omega)}{\varepsilon_0} = \lambda[\theta(\omega-\omega_1) - \theta(\omega-\omega_2)], \quad \omega_2 > \omega_1 > 0 \qquad (1)$$

$$\mathrm{Re}\,\frac{\varepsilon(\omega)}{\varepsilon_0} = 1 + \frac{2}{\pi} P\int_0^\infty \frac{\omega'\,\mathrm{Im}\,\varepsilon(\omega')/\varepsilon_0}{\omega'^2-\omega^2}\,d\omega' \qquad (2)$$

$$= 1 + \frac{2}{\pi}\left[P\int_{\omega_1}^\infty \frac{\lambda\omega'}{\omega'^2-\omega^2}\,d\omega' - P\int_{\omega_2}^\infty \frac{\lambda\omega'}{\omega'^2-\omega^2}\,d\omega'\right]$$

$$= 1 + \frac{2}{\pi}\lambda P\int_{\omega_1}^{\omega_2} \frac{\omega'}{\omega'^2-\omega^2}\,d\omega'$$

$$= 1 + \frac{2}{\pi}\lambda \left.\frac{\ln(\omega'^2-\omega^2)}{2}\right|_{\omega_1}^{\omega_2}$$

$$\mathrm{Re}\,\frac{\varepsilon(\omega)}{\varepsilon_0} = 1 + \frac{\lambda}{\pi}\ln\left(\frac{\omega_2^2-\omega^2}{\omega_1^2-\omega^2}\right) \quad \# \qquad (3)$$

b) $$\mathrm{Im}\,\frac{\varepsilon(\omega)}{\varepsilon_0} = \frac{\gamma\lambda\omega}{(\omega_0^2-\omega^2)^2+\gamma^2\omega^2} \qquad (4)$$

$$= \frac{\gamma\lambda\omega}{[(\omega_0^2-\omega^2)+i\gamma\omega][(\omega_0^2-\omega^2)-i\gamma\omega]}$$

$$= \frac{\gamma\lambda\omega}{(\omega-\omega_+)(\omega-\omega_+^*)(\omega-\omega_-)(\omega-\omega_-^*)} \qquad (5)$$

where $$\omega_\pm = -\frac{i\gamma}{2} \pm \sqrt{\omega_0^2 - \frac{\gamma^2}{4}}$$

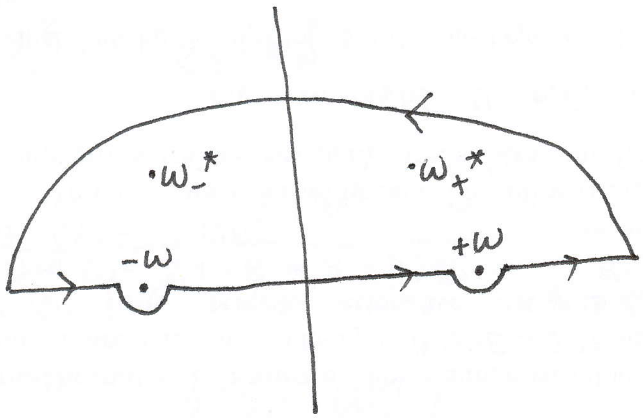
$$\omega_\pm^* = \frac{i\gamma}{2} \pm \sqrt{\omega_0^2 - \gamma^2/4}$$

(5) → (2):

$$\mathrm{Re}\,\frac{\varepsilon(\omega)}{\varepsilon_0} = 1 + \frac{2}{\pi} P\int_0^{\infty} \frac{\gamma\Lambda\omega'^2\, d\omega'}{(\omega'^2-\omega^2)(\omega'-\omega_+)(\omega'-\omega_+^*)(\omega'-\omega_-)(\omega'-\omega_-^*)}$$

$$= 1 + \frac{\gamma\Lambda}{\pi} P\int_{-\infty}^{+\infty} \frac{\omega'^2\, d\omega'}{(\omega'^2-\omega^2)(\omega'-\omega_+)(\omega'-\omega_+^*)(\omega'-\omega_-)(\omega'-\omega_-^*)} \qquad (6)$$

to evaluate the integral, I pick the following contour:



thus,

$$P[\ \ ] = 2\pi i\left[\mathrm{Res}(\omega_-^*) + \mathrm{Res}(\omega_+^*)\right] + \pi i\left[\mathrm{Res}(-\omega) + \mathrm{Res}(\omega)\right] \qquad (7)$$

$$\mathrm{Res}(-\omega) = \frac{\omega^2}{-2\omega(-\omega-\omega_+)(-\omega-\omega_+^*)(-\omega-\omega_-)(-\omega-\omega_-^*)}$$

$$\mathrm{Res}(\omega) = \frac{\omega^2}{2\omega(\omega-\omega_+)(\omega-\omega_+^*)(\omega-\omega_-)(\omega-\omega_-^*)}$$

$$= -\mathrm{Res}(-\omega) \qquad (8)$$

$$\mathrm{Res}(\omega_+^*) = \frac{\omega_+^{*2}}{(\omega_+^{*2}-\omega^2)(\omega_+^*-\omega_+)(\omega_+^*-\omega_-)(\omega_+^*-\omega_-^*)}$$

$$= \frac{-\frac{\gamma^2}{2} + \omega_0^2 + i\gamma\sqrt{\omega_0^2-\gamma^2/4}}{\left(-\frac{\gamma^2}{2}+\omega_0^2+i\gamma\sqrt{\omega_0^2-\gamma^2/4}-\omega^2\right) i\gamma\left(i\gamma+2\sqrt{\omega_0^2-\gamma^2/4}\right) 2\sqrt{\omega_0^2-\gamma^2/4}}$$

$$\mathrm{Res}(\omega_-^*) = \frac{\omega_-^{*2}}{(\omega_-^{*2}-\omega^2)(\omega_-^*-\omega_+)(\omega_-^*-\omega_+^*)(\omega_-^*-\omega_-)}$$

$$= \frac{-\frac{\gamma^2}{2} + \omega_0^2 - i\gamma\sqrt{\omega_0^2-\gamma^2/4}}{\left(-\frac{\gamma^2}{2}+\omega_0^2 - i\gamma\sqrt{\omega_0^2-\gamma^2/4} - \omega^2\right) i\gamma\left(i\gamma - 2\sqrt{\omega_0^2-\gamma^2/4}\right)\left(-2\sqrt{\omega_0^2-\gamma^2/4}\right)}$$

$\mathrm{Res}(\omega_+^*) + \mathrm{Res}(\omega_-^*)$

$$= \frac{2(\omega^2-\omega_0^2)}{-i\gamma\left(\gamma^2+2\omega^2-2\omega_0^2 - i\gamma\sqrt{4\omega_0^2-\gamma^2}\right)\left(\gamma^2+2\omega^2-2\omega_0^2+i\gamma\sqrt{4\omega_0^2-\gamma^2}\right)}$$

$$= \frac{2(\omega^2-\omega_0^2)}{-i\gamma\left([\gamma^2+2\omega^2-2\omega_0^2]^2 + \gamma^2(4\omega_0^2-\gamma^2)\right)}$$

$$= \frac{2(\omega^2-\omega_0^2)}{-i\gamma\left[4(\omega^4+\omega_0^4-2\omega^2\omega_0^2)+4\gamma^2\omega^2\right]}$$

$$= \frac{(\omega^2-\omega_0^2)}{-2i\gamma\left[(\omega^2-\omega_0^2)^2+\gamma^2\omega^2\right]} \qquad (9)$$

$(8),(9) \rightarrow (7)$:

$$P[\quad] = \frac{2\pi i\,(\omega^2-\omega_0^2)}{-2i\gamma\left[(\omega^2-\omega_0^2)^2+\gamma^2\omega^2\right]}$$

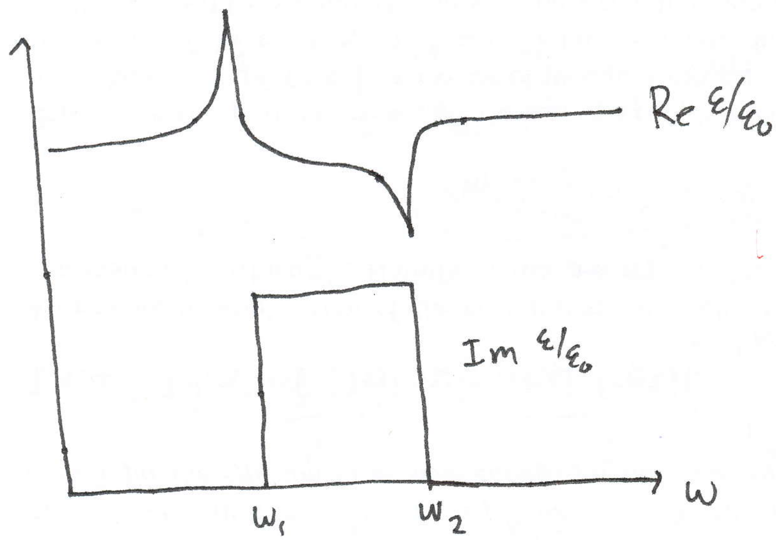
$$= -\frac{\pi}{\gamma}\frac{(\omega^2-\omega_0^2)}{\left[(\omega^2-\omega_0^2)^2+\gamma^2\omega^2\right]} \qquad (10)$$

(10) → (6) :

$$\mathrm{Re}\,\frac{\varepsilon(\omega)}{\varepsilon_0} = 1 + \frac{\lambda\,(\omega_0^2 - \omega^2)}{(\omega^2-\omega_0^2)^2 + \gamma^2\omega^2} \qquad (11)$$

#

Part (a)



Part (b)