1. Jackson 6.1

a) $\psi(\vec{x},t) = \int \frac{[f(\vec{x},t)]_{ret}}{|\vec{x}-\vec{x}'|} d^3x'$

$$= \int dz' \int dy' \int dx' \frac{\delta(x')\delta(y')\delta(t - \frac{1}{c}[(x-x')^2 + (y-y')^2 + (z-z')^2]^{1/2})}{[(x-x')^2 + (y-y')^2 + (z-z')^2]^{1/2}}$$

$$= \int dz' \frac{\delta(t - \frac{1}{c}[\rho^2 + (z-z')^2]^{1/2})}{(\rho^2 + (z-z')^2)^{1/2}} , \rho^2 = x^2 + y^2 \quad (1)$$

Note that: $\delta(f(x)) = \sum_i \frac{1}{\left|\frac{\partial f}{\partial x}\right|_{x_i}} \delta(x - x_i)$, $f(x_i) = 0$

$f(z') = t - \frac{1}{c}(\rho^2 + (z-z')^2)^{1/2}$

$f(z') = 0$ when $z' = z \pm (t^2c^2 - \rho^2)^{1/2}$, $tc \geq \rho$

$\frac{df}{dz'} = \frac{(z-z')}{c(\rho^2 + (z-z')^2)^{1/2}}$

$\left|\frac{\partial f}{\partial z'}\right|_{z \pm (t^2c^2-\rho^2)^{1/2}} = \frac{(t^2c^2 - \rho^2)^{1/2}}{c^2 t}$, $tc \geq \rho$

$\Rightarrow$

$$\delta(f(z')) = \frac{c^2 t}{(t^2c^2-\rho^2)^{1/2}} \left\{ \delta(z' - [z + (t^2c^2-\rho^2)^{1/2}]) + \delta(z' - [z - (t^2c^2-\rho^2)^{1/2}]) \right\} , tc \geq \rho \quad (2)$$

(2) $\Rightarrow$ (1):

$$\psi(\vec{x},t) = \frac{c^2 t}{(t^2c^2-\rho^2)^{1/2}} \int \frac{dz'}{[\rho^2 + (z-z')^2]^{1/2}} \left\{ \delta(z' - [z + (t^2c^2-\rho^2)^{1/2}]) + \delta(z' - [z - (t^2c^2-\rho^2)^{1/2}]) \right\}$$

$$= \frac{c^2 t}{(t^2c^2-\rho^2)^{1/2}} \left\{ \frac{1}{[\rho^2 + (t^2c^2-\rho^2)]^{1/2}} + \frac{1}{[\rho^2 + (t^2c^2-\rho^2)]^{1/2}} \right\} , tc \geq \rho$$

$$= \frac{2c}{(t^2c^2-\rho^2)^{1/2}} , tc \geq \rho$$

$$\psi(\vec{x},t) = \frac{2c\,\Theta(ct-\rho)}{(t^2c^2-\rho^2)^{1/2}} \quad \#$$

b) $\psi(\vec{x},t) = \int \frac{[f(\vec{x}',t)]_{ret}}{|\vec{x}-\vec{x}'|} d^3x'$

$$= \int dx' \int dy' \int dz' \, \frac{\delta(z')\,\delta(t - 1/c\,[(x-x')^2 + (y-y')^2 + (z-z')^2]^{1/2})}{[(x-x')^2 + (y-y')^2 + (z-z')^2]^{1/2}}$$

$$= \int dx' \int dy' \, \frac{\delta(t - 1/c\,[(x-x')^2 + (y-y')^2 + z^2]^{1/2})}{[(x-x')^2 + (y-y')^2 + z^2]^{1/2}}$$

$$= \int dx \int dy \, \frac{\delta(t - 1/c\,[x^2 + y^2 + z^2]^{1/2})}{(x^2 + y^2 + z^2)^{1/2}} \quad , \; x' \ni x - x' \;\&\; y' \ni y - y'$$

$$= \int_0^{2\pi} d\theta \int_0^{\infty} \rho\, d\rho \, \frac{\delta(t - 1/c\,(\rho^2 + z^2)^{1/2})}{(\rho^2 + z^2)^{1/2}} \quad , \; x = \rho\cos\theta \;\&\; y = \rho\sin\theta$$

$$= 2\pi \int_0^{\infty} d\rho \, \frac{\rho}{(\rho^2 + z^2)^{1/2}} \, \delta(t - 1/c\,[\rho^2 + z^2]^{1/2}) \qquad (3)$$

Note that: $t - 1/c\,[\rho^2 + z^2]^{1/2} = 0 \Rightarrow \rho = \sqrt{c^2t^2 - z^2} \quad , \; ct \geq |z|$

thus, $\delta(t - 1/c\,[\rho^2 + z^2]^{1/2}) = \frac{c^2 t}{(c^2t^2 - z^2)^{1/2}} \delta(\rho - \sqrt{c^2t^2 - z^2})\,\Theta(ct - |z|) \qquad (4)$

(4) $\ni$ (3):

$$\psi(\vec{x},t) = \frac{2\pi c^2 t}{(c^2t^2 - z^2)^{1/2}} \int_0^{\infty} d\rho \, \frac{\rho}{(\rho^2 + z^2)^{1/2}} \, \delta(\rho - \sqrt{c^2t^2 - z^2})\,\Theta(ct - |z|)$$

$$= \frac{2\pi c^2 t}{(c^2t^2 - z^2)^{1/2}} \; \frac{(c^2t^2 - z^2)^{1/2}}{ct} \; \Theta(ct - |z|)$$

$$\psi(\vec{x},t) = 2\pi c \, \Theta(ct - |z|)$$

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Note that I'm using z-axis instead of x-axis.