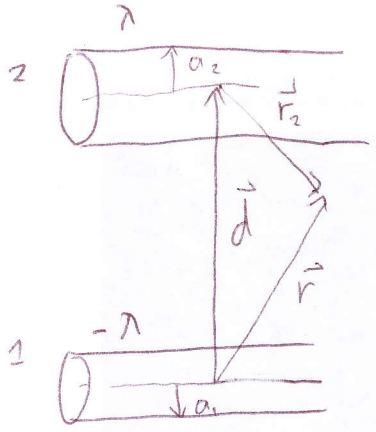
1.7)



$$\vec{E}_2 = \frac{\lambda}{2\pi\varepsilon_0}\frac{\vec{r}_2}{r_2^2} = \frac{\lambda}{2\pi\varepsilon_0}\frac{(\vec{r}-\vec{d})}{|\vec{r}-\vec{d}|^2}$$

$$\vec{E}_1 = -\frac{\lambda}{2\pi\varepsilon_0}\frac{\vec{r}}{r^2}$$

a) $$\vec{E} = \vec{E}_1 + \vec{E}_2 = -\frac{\lambda}{2\pi\varepsilon_0}\left(\frac{\vec{r}}{r^2} + \frac{(\vec{d}-\vec{r})}{|\vec{d}-\vec{r}|^2}\right)$$

$$\Delta V = -\int_1^2 \vec{E}\cdot d\vec{\ell}$$

let's integrate along $\vec{d}$:

$$\Delta V = \frac{\lambda}{2\pi\varepsilon_0}\int_{a_1}^{d-a_2}\left(\frac{1}{r} + \frac{1}{d-r}\right)dr$$

$$= \frac{\lambda}{2\pi\varepsilon_0}\left(\ln\left(\frac{d-a_2}{a_1}\right) + \ln\left(\frac{d-a_1}{a_2}\right)\right)$$

$$= \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{(d-a_2)(d-a_1)}{a_1 a_2}\right)$$

$$\Delta V \simeq \frac{\lambda}{\pi\varepsilon_0}\ln\left(\frac{d}{\sqrt{a_1 a_2}}\right), \text{ For } d \gg a_1, a_2$$

$$C = \frac{Q}{\Delta V} = \frac{\lambda\ell}{\lambda/\pi\varepsilon_0}\ln\left(\frac{d}{\sqrt{a_1 a_2}}\right)^{-1} \Rightarrow \frac{C}{\ell} = \pi\varepsilon_0\left[\ln\left(\frac{d}{\sqrt{a_1 a_2}}\right)\right]^{-1}$$

b) For $a_1 = a_2 \equiv a$; $a = d\,e^{-\pi\varepsilon_0/C}$

$a \simeq 0.1\,d$, For $C = 1.2\times10^{-11}$ F/m

d	diameter
0.5cm	1mm
1.5cm	3mm
50cm	10mm