30/30 very nice

1) Jackson 1.10

$$\phi(\vec{x}) = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho(\vec{x}\,')}{R}\,d^3x' + \frac{1}{4\pi}\oint_S \left[\frac{1}{R}\frac{\partial\phi}{\partial n'} - \phi(\vec{x}\,')\frac{\partial}{\partial n'}\left(\frac{1}{R}\right)\right] da'$$

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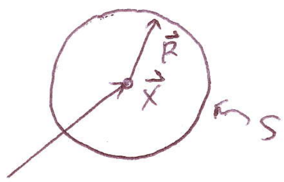
For a charge free region : $\rho = 0$

thus,

$$\phi(\vec{x}) = \frac{1}{4\pi}\oint_S \left[\frac{1}{R}\frac{\partial\phi}{\partial n'} - \phi(\vec{x}\,')\frac{\partial}{\partial n'}\left(\frac{1}{R}\right)\right] da'$$

let S be a sphere with radius R and center at $\vec{x}$.



$$\phi(\vec{x}) = \frac{1}{4\pi R}\oint_S \vec{\nabla}\phi\cdot d\vec{a}\,' - \frac{1}{4\pi}\oint_S \phi(\vec{x}\,')\,\vec{\nabla}\left(\frac{1}{R}\right)\cdot d\vec{a}\,'$$

From Divergence theorem:

$$\oint_S \vec{\nabla}\phi\cdot d\vec{a}\,' = \int_V \vec{\nabla}\cdot(\vec{\nabla}\phi)\,d^3x' = -\int_V \vec{\nabla}\cdot\vec{E}\,d^3x' = 0$$

since $\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0 = 0$.

$$\phi(\vec{x}) = -\frac{1}{4\pi}\oint_S \phi(\vec{x}\,')\left(-\frac{\hat{R}}{R^2}\right)\cdot d\vec{a}\,'$$

not what I had in mind but ok

$$= \frac{1}{4\pi R^2}\oint_S \phi(\vec{x}\,')\,da'$$

$\Rightarrow$ Mean Value Theorem.